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**PROCEDURES FOR SHAPE OPTIMIZATION OF
GAS TURBINE DISKS**

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ABSTRACT

Two procedures, the feasible direction method and sequential linear programming, for shape optimization of gas turbine disks are presented in this paper. The objective of these procedures is to obtain optimal designs of turbine disks with geometric and stress constraints. The coordinates of the selected points on the disk contours are used as the design variables. Structural weight, stress and their derivatives with respect to the design variables are calculated by an efficient finite-element method for design sensitivity analysis. Numerical examples of the optimal designs of a disk subjected to thermo-mechanical loadings are presented to illustrate and compare the effectiveness of these two procedures.

1. INTRODUCTION

The problem of how to efficiently minimize the weight of a gas turbine engine disk while satisfying the stress design requirement and keeping the disk size within a prescribed geometric envelope is an important topic in the gas turbine industry. A stress function in common use is the Von Mises stress. Therefore, a requirement can be to limit the value of the von Mises stress anywhere on the disk to a prescribed value.

This task becomes one of shape optimization of an axisymmetric structure with the objective of minimizing the weight while meeting the geometric and the stress constraints. Each of the procedures involved requires a solver and an optimizer. The solver provides weight, stress and their derivatives with respect to the design variables. An optimizer must be selected which can effectively utilize the solver.

It has been shown that the weight and stress gradients can be obtained directly from a finite-element program [1-5]. In this paper, two optimization procedures which can effectively utilize the weight and stress gradients are proposed for disk shape optimization. These two methods are the feasible direction method [6] and sequential linear programming [1,4,7].

In structural optimization, design variables may be finite-element nodal coordinates, element thickness, etc. The derivatives of the objective function and the constraint functions with respect to the design variables provide the variational trends of the structures for optimization. Calculation of these derivatives is known as design sensitivity analysis.

In this paper, an efficient method is used for design sensitivity analysis [5]. The technique of isoparametric mapping is used to generate a finite-element mesh from a small set of master nodes. In order to assure that a general boundary shape can be achieved for the optimal design of a complex structural shape, selected coordinates of the master nodes are used as the design variables. These variables are permitted to change within a specified design envelope.

The formulations and computational results of these two optimization procedures are presented in the following sections.

2. FORMULATION

2.1 PROBLEM STATEMENT

The general statement of the problem to be dealt with in this paper is to minimize an objective function

$$F = f(A) \quad (1)$$

while satisfying the constraints

$$g_j(A) \leq 0 \quad j = 1, \dots, M \quad (2)$$

where the N dimensional hyperspace design point, A , is a vector of design variables initially lying in the feasible region with side constraints

$$a_i^l \leq a_i \leq a_i^u \quad i = 1, \dots, N \quad (3)$$

In this paper, the objective function is W , the structural weight, the constraint functions are the structural response such as σ , the stress, and the design variables are the coordinates of the selected points on the structural contours with upper and lower constraints a_i^u and a_i^l .

2.2 FEASIBLE DIRECTION METHOD

The feasible direction method efficiently uses weight and constraint gradients. This method requires two operational phases. The first one is the steepest decent phase (SD) which requires only the weight gradients, the second one is a linear programming phase (LP) which requires both the weight and constraint gradients.

Starting with a feasible design point, A^0 , a better point

$$A = A^0 + \alpha S \quad (4)$$

can be achieved by moving A^0 in the feasible region a distance of α in the direction S .

The feasible region is bounded by the constraint limits with the constraint margins, e_j , as shown in Figure 1. The boundary zones which are thus formed by the hypersurfaces parallel to the constraint limit hypersurfaces are known as the LP region. The rest of the feasible space is known as the SD region. The design points within either regions are the feasible design points.

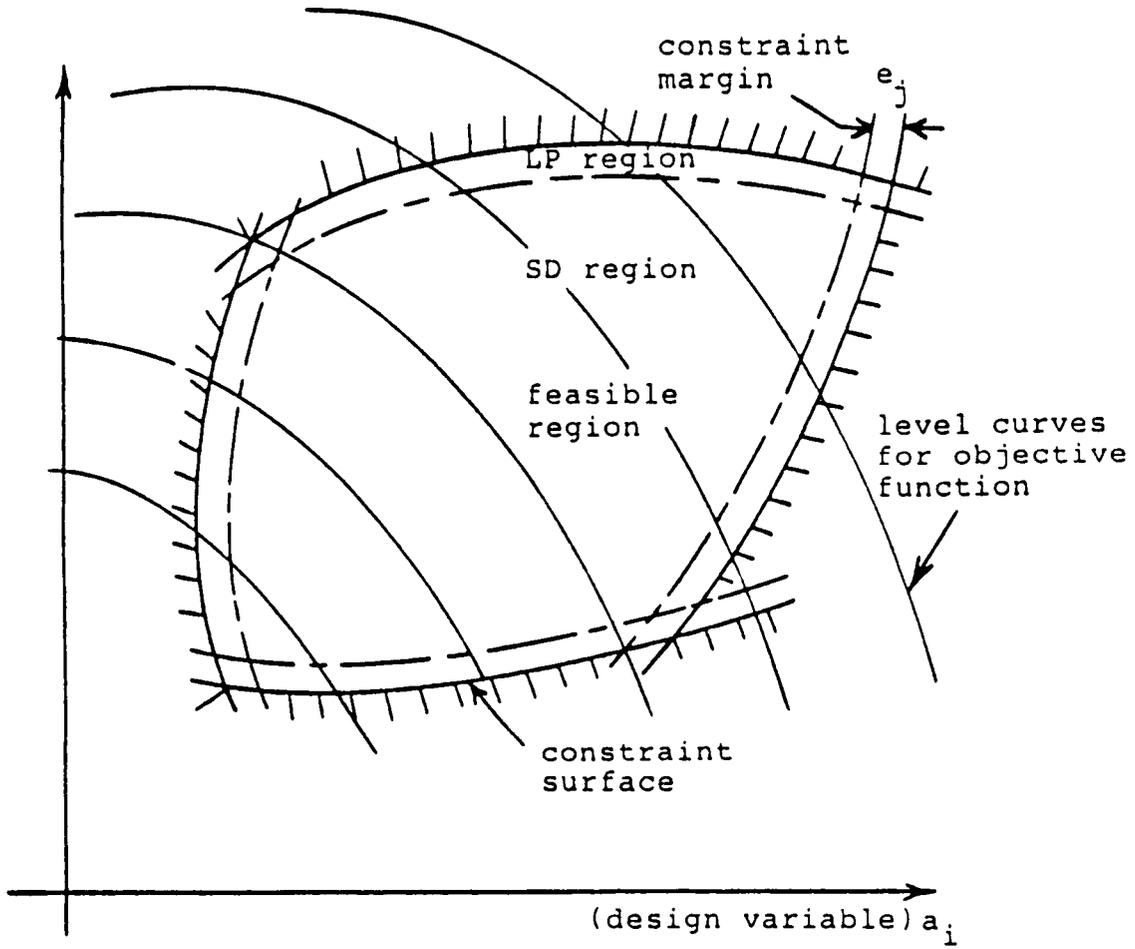


Figure 1. Design space for feasible direction method.

In the SD region S is the negative of the weight gradients, while in the LP region S is calculated by a linear programming procedure, [8], with the formulations

$$S^T \nabla g_j(A) + \theta_j \beta \leq \sum_i \frac{\partial g_j}{\partial a_i} \quad j = 1, \dots, M \quad (5)$$

$$S^T \nabla f(A) + \beta \leq \sum_i \frac{\partial f}{\partial a_i} \quad (6)$$

and side constraints on directions are

$$s_i \leq 2 \quad i = 1, \dots, N \quad (7)$$

$$s_i \geq 0 \quad i = 1, \dots, N \quad (8)$$

where θ_j are the pushoff factors and β is the objective function.

After a feasible direction to proceed is found, the distance to travel in that direction is calculated by using a Powell's univariate search. The minimum distance to be traveled is 0. The maximum distance is taken to be the maximum of all the permissible variation of each design variable in the feasible region. The actual distance will occur at any point along the line between 0 and the maximum distance point.

Before engaging in Powell's search, the finite-element method is used to check if a negative determinant of Jacobi transformation matrix occurs at any Gauss integration point [9]. If a design point yields a negative value, then the design point is moved back along the line by a specified fraction. If a negative value still results, the process is repeated until a positive value occurs. This positive value is taken as the maximum distance for a Powell's univariate search. If the new design point is in the feasible region, a new direction and univariate search will be made again for the next iteration. If this new point is in the infeasible region, then an interpolation scheme is used to bring the design point back into the feasible region [6]. If the geometric constraints are violated, a linear interpolation routine is used. If the stress constraint is violated, a quadratic interpolation procedure is used.

After a new feasible design point is found, the process is repeated until the convergence criteria are satisfied.

2.3 SEQUENTIAL LINEAR PROGRAMMING

The sequential linear programming procedure linearizes the nonlinear objective and constraint functions within a specified range where the linear programming procedure will be applied repeatedly. This method efficiently utilizes the gradients of the objective and the constraint functions and has been shown to be reliable in many different applications.

The structural weight and responses generally are nonlinear functions of the design variables. Using a first order Taylor series expansion centered at the current design, these functions can be approximated by

$$W = W^0 + \sum_i \frac{\partial W}{\partial a_i} (a_i - a_i^0) \quad (9)$$

$$\sigma = \sigma^0 + \sum_i \frac{\partial \sigma}{\partial a_i} (a_i - a_i^0) \quad (10)$$

where the superscript 0 denotes the current design.

These equations are used to form a linear programming problem

$$- \sum_i \frac{\partial W}{\partial a_i} a_i = C = W^0 - W - \sum_i \frac{\partial W}{\partial a_i} a_i^0 \quad (11)$$

$$\sum_i \frac{\partial \sigma}{\partial a_i} a_i \leq \sigma_d - \sigma^0 + \sum_i \frac{\partial \sigma}{\partial a_i} a_i^0 \quad (12)$$

and the side constraints on design variables

$$a_i \leq a_i^0 (1 + \Delta) \quad (13)$$

$$-a_i \leq -a_i^0 (1 - \Delta) \quad (14)$$

where

$$\Delta = \text{Max} (\Delta_{\min}, \Delta_c)$$

$$\Delta_i = \text{Min} \left(\left| \frac{\sigma_d - \sigma^0}{\frac{\partial \sigma}{\partial a_i}} \right|, \Delta_{\max} \right)$$

$$\Delta_c = \text{Max} (\Delta_i) \quad \text{for the beginning of each LP iteration}$$

$$\Delta_c = r \cdot \Delta \quad \text{for the subsequent LP iteration}$$

and r is the step size reduction factor.

The linear programming problem is solved by a revised simplex method [8]. After a new design is found, the process is repeated until the convergence criteria are satisfied.

3. Numerical Examples

Shape optimization of an actual gas turbine disk subjected to thermo-mechanical loadings is used as an illustration. The flow chart of the design optimization process is shown in Figure 2.

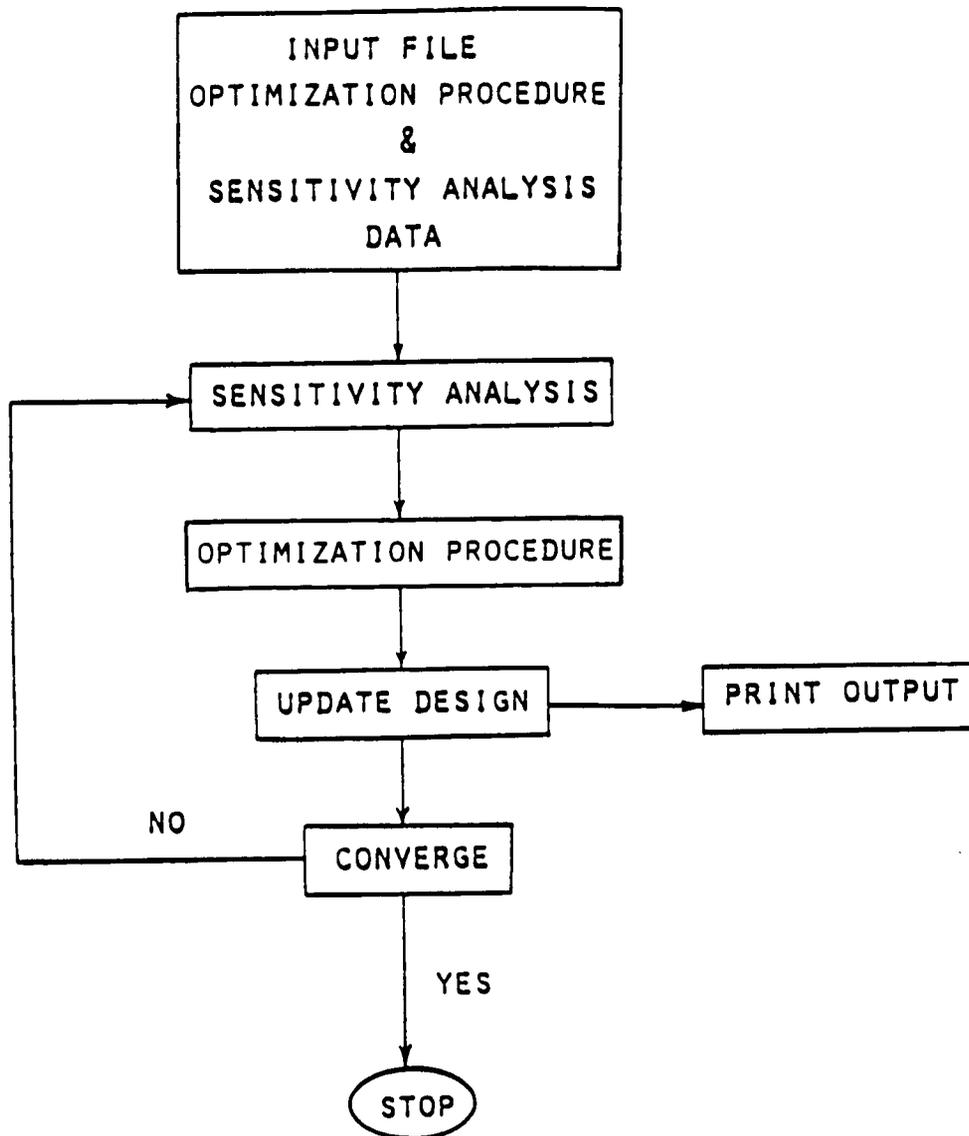


Figure 2. Disk design optimization system flow chart

The master finite-element mesh and the generated finite-element mesh are shown in Figures 3a and 3b. Seventy master nodes are used to form 38 4-noded linear master elements for the disk. Both radial and axial coordinates of master nodes numbered from 1 through 30 are used as design variables. The finite-element mesh generated from the master finite-element mesh has 130 8-noded quadratic elements and 469 nodes.

This disk has a uniform initial temperature of 70° F. The radial coordinates at the hub and the tip of the disk are 0.8 inch and 4.85 inches, respectively. The operating temperatures of the disk are set arbitrarily to vary linearly from 80° F at the hub to 485° F at the tip.

The disk rotates at a constant speed of 22,000 rpm. It has a distributed load of 24,000 psi acting radially outward on the tip circumferential surface. Axial coordinates of the points on side A-B are fixed as boundary conditions. The yield stress of the disk is specified at 125,000 psi. The maximum Von Mises stress is used as the stress design criterion.

Feasible direction method and sequential linear programming are used in the first and the second examples, respectively. The computer software developed was executed on the IBM 3090 using a VS 2.2 compiler. The optimization procedures are considered to have converged if the change of the structural weight is less than 0.1% for the two successive iterations. The computational results are listed in Tables 1 and 2.

In the first example, the feasible direction method is used. Convergence is achieved in 8 iterations with 216 CPU seconds of computational time. The total weight is reduced from 18.0 pounds to 13.978 pounds. The maximum Von Mises stress is increased from 91,914 psi to 124,668 psi.

In the second example, the sequential linear programming is used. Convergence is achieved in 9 iterations with 369 CPU seconds of computational time. The total weight is reduced from 18.0 pounds to 13.873 pounds. The maximum Von Mises stress is increased from 91,914 psi to 124,999 psi.

The optimal designs of both examples satisfy the stress design criterion. However, slightly different weight reductions, 22.35% and 22.93%, respectively, are achieved for the first and the second examples. For the same convergence criterion the first example requires about 40% less computational time than the second example. However, for the same percentage of weight reduction both methods require about the same computational time.

The shapes of the optimal designs of the two examples are almost identical. The shape of the optimal design obtained with the sequential linear programming is shown in Figure 4.

The computational results indicate that by using the procedures developed, shape optimization of gas turbine disks with complicated

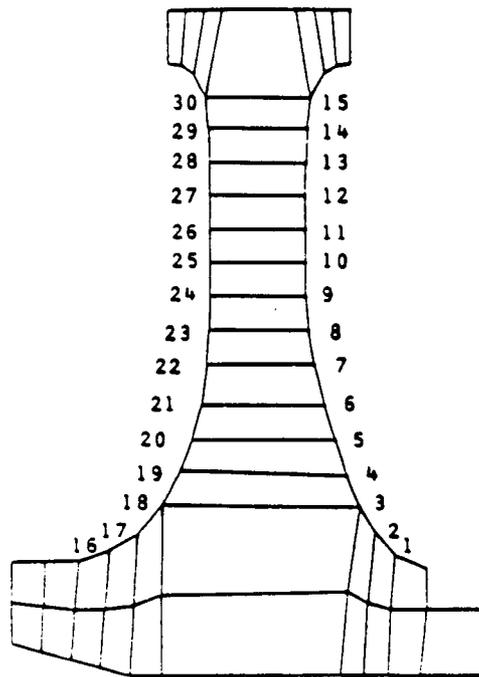


Figure 3a. Master finite-element model for the initial disk design

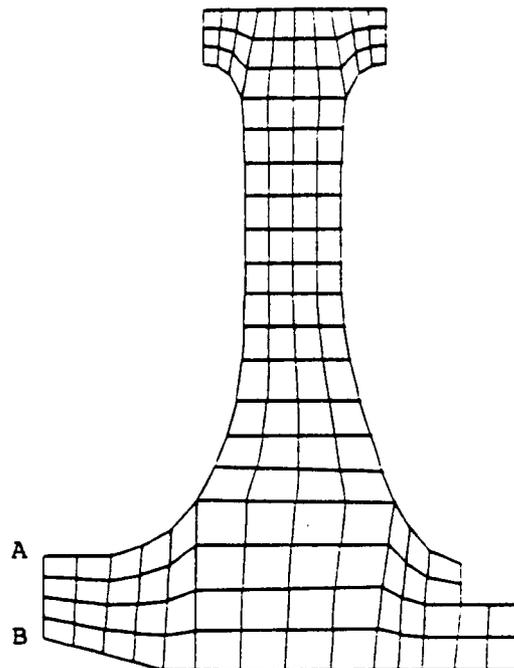


Figure 3b. Generated finite-element model for the initial disk design

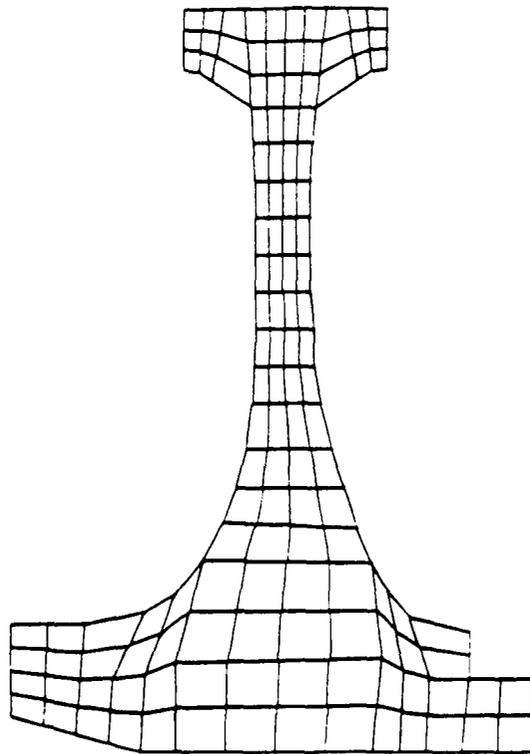


Figure 4. Optimal disk design

Table 1. Computation results for example 1.

Iteration No.	Iteration Type	Max. Von Mises Stress (psi)	Weight lb	Successive Reduction (%)	Total Reduction (%)
0	Original	91914	18.000		
1	SD	91478	16.943	5.869	5.869
2	SD	97185	15.872	6.322	11.820
3	SD	110374	14.784	6.856	17.865
4	SD	116609	14.350	2.937	20.278
5	SD	122822	14.027	2.249	22.071
6	LP	123243	14.000	0.195	22.223
7	LP	124921	13.979	0.149	22.339
8	LP	124688	13.978	0.008	22.345

Table 2. Computation results for example 2.

Iteration No.	Max. Von Mises Stress (psi)	Weight lb	Successive Reduction (%)	Total Reduction (%)
0	91914	18.000		
1	110802	15.012	16.602	16.602
2	121876	14.287	4.824	20.625
3	124964	13.918	2.586	22.678
4	124755	13.896	0.161	22.802
5	124989	13.884	0.086	22.868
6	124994	13.878	0.042	22.901
7	124997	13.875	0.020	22.916
8	124999	13.874	0.011	22.924
9	124999	13.873	0.006	22.929

contour shapes and loading conditions can be achieved with relatively short computational time.

REFERENCES

1. Zienkiewicz, O.C.; and Campbell, J.S.: 'Shape Optimization and Sequential Linear Programming', Optimum Structural Design (Edited by Gallagher R.H. and Zienkiewicz O.C.), Wiley, New York, 1973
2. Ramakrishnan, C.V.; and Francavilla, A.: 'Structural Shape Optimization Using Penalty Functions', J. Struc. Mech., 3(4), pp. 403-422, 1974-75.
3. Botkin, M.E.: 'Shape Optimization of Plate and Shell Structures', AIAA J., Vol. 20, pp. 268-273, 1981.
4. Wang, S.Y.; Sun, Y.; and Gallagher, R.H.: 'Sensitivity Analysis in Shape Optimization of Continuum Structures', Comp. Struc., Vol. 20, pp. 855-867, 1985.
5. Cheu, T.C.: 'Sensitivity Analysis and Shape Optimization of Axisymmetric Structures', Vol. 28, pp. 95-108, January 1989.
6. Fox, R.L.: Optimization Methods for Engineering Design, Addison-Wesley, London, 1970.

7. Pederson, P.: 'The Integrated Approach of FEM-SLP for Solving Problems of Optimal Designs', Optimization of Distributed Parameter Structures, (Edited by Hang and Cea) Sijthoff and Nourdhoff, pp. 757-780, 1981.
8. Hanson, R.J.; and Wisniewski, J.A.: 'A Revised Simplex Code for LP Problems Using Orthogonal Decomposition - A User's Guide', Sandia Lab. Report Sand 78-2322, 1979.
9. Zienkiewicz, O.C.: The Finite Element Method, 3rd edition, McGraw-Hill, New York, 1977.